Multiparticle phase shift analysis in peripheral production experiments: Lessons from the Past

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Abstract
We recall some of our understanding of meson resonance production in high energy collisions from 20 years ago and its implications for a new generation of high statistics experiments. We discuss S-matrix theory, duality and Regge exchanges. We briefly mention some computing issues.

Introduction
Some 20 years ago, I switched from particle physics to largely working in computer science. At that time, I was part of teams completing analysis of three experiments at Fermilab; one [1] E260 saw hadron jets at "high" transverse momentum (perhaps 5 GeV/c); another [2] E350 studied triple-Regge theory and measured the ρ and A_2 trajectories out to $-t = 2\ (\mathrm{GeV/c})^2$; a third [3] E110 studied meson resonance production at 50 to 200 GeV/c with π^- and K^- beams. Progress has of course been dramatic in the area of the first experiment with results in general agreement with QCD now seen up to some 50 GeV/c transverse jet momentum. In the final two experiments, we were studying the soft limit of strong interactions. Here there has been some progress in understanding resonance structure – both from high statistics experiments and more importantly major advances in understanding quark model spectroscopy. However progress in understanding peripheral production has been limited except for diffractive reactions where the Pomeron phenomenology [4] has advanced based on deep inelastic diffractive data. In reading this article, please note I am recalling understandings that I had some 20 years ago. I apologize for any mistakes.

Some Partial Wave Analysis Issues
In examining resonance M* production in reactions like $\gamma\, p \rightarrow M^*\, p$, one can study both the resonance's parameters (including its existence) and its production characteristics. Currently we have no profound theoretical understanding of production dynamics but do have a deeper understanding and interest in meson spectroscopy and so the latter is our primary goal. Nevertheless good phenomenology of the production dynamics appears essential, as it is critical in the formulation of the partial wave analysis (PWA). These intrinsically parameterize amplitudes and the spin, phase and baryon vertex structure of the amplitudes is strongly affected by the production dynamics. This implies that PWA is technically hard and it is very difficult to produce results with minimum and perhaps more importantly understandable bias.

Study of resonances in electron-positron collisions $e^+ e^- \rightarrow M^*$ is clean compared to the hadron case but suffers from both limited statistics and selection rules that imply that many important resonances cannot be produced in this fashion. Study of M* in $e^+ e^- \rightarrow$ M* X does not have selection rule restrictions but has many of the obfuscatory effects of the hadron case compounded by modest statistics. Thus for much spectroscopy, study of it via production in Baryon-Baryon and Photon-Baryon collisions is attractive.

We can divide such spectroscopy in three broad classes of increasing analysis complexity:

- **2 ⇨ 2 reactions:** These are clearly very clean but limited to a few well known reactions such as π N ⇨ π N , γ N ⇨ π N and π N ⇨ η N. Correspondingly this approach can only tackle baryon resonances. Analyses of 2 → 2 reactions with good enough statistics are largely uncontroversial at the partial wave analysis level. However the interpretation in terms of resonances may be impossible if they have small couplings to the two-body channels.
- **2 ⇨ 3 or more final particle reactions where we are interested in resonances decaying to full final state:** Examples are π N ⇨ π π N, $\bar{p}\,p$ annihilation etc. (in this case, one is looking at M*X but with well defined s-channel quantum numbers) where the full final state is partial wave analyzed. Here we have several new technical complications such as a large increase in the number of degrees of freedom. In particular realistic ways approaches to 3 particle final states usually require resonance assumptions which have intrinsic uncertainties. There are some dynamical effects (such as π exchange) which can not be studied in π N ⇨ π N and so their effect on the PWA is unclear. They could be serious as π exchange populates higher partial waves than the exchanges seen in π N ⇨ π N.
- **2 ⇨(state to be analyzed) plus one or more other particles (peripherally produced)** – this in particular includes case of peripheral production of meson states by π and γ beams. Here there are a further set of problems due to confusion of "other processes apart from resonance of interest" forming full final state. Further it is not very easy to unambiguously define amplitudes to be analyzed. The latter remark assumes the very large number of degrees of freedom precludes solving all these problems by a general parameterization and "just fitting".

The latter difficult case is the one we are interested in! We comment on some of the issues in the following.

S-Matrix Unitarity and Phenomenology

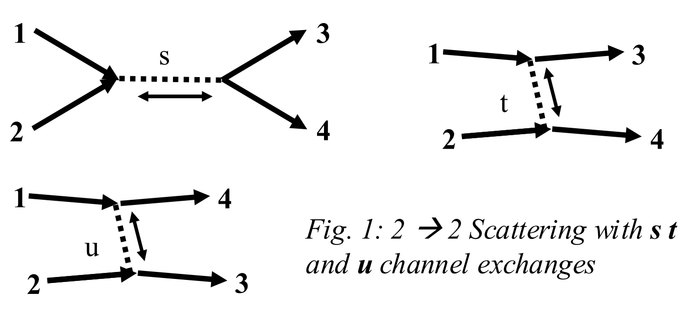


*Fig. 1: 2 → 2 Scattering with **s t** and **u** channel exchanges*

The S-Matrix approach was very popular 30 years ago and established a set of "true principles" which appeared to be very strong constraints on the scattering amplitude. The dream that these constraints could be strong enough to determine the amplitude proved impossible to turn into reality. Further no way was discovered to guarantee that known "truths" were satisfied. For example we understood as in figure 1, that a single analytic function described the three reactions of the s t and u channels. The resonances in the s channel were "created" by the forces in the t and u channel whose resonances become Regge Poles and not traditional (fixed) poles when viewed as exchanged particles or forces. The Veneziano model gave an elegant sample amplitude (see paper by Donnachie in these proceedings) illustrating how the Regge Poles (resonances) in one channel are not added to the exchange poles but rather they are different ways at looking at the same amplitude. This "duality" is clearest for mesons like the A_1 produced in reactions like $\pi^- p \Rightarrow \pi^- \pi^- \pi^+ p$. As shown in figure 2, the meson production (figure 2(a)) and the π exchange (figure 2(b)) descriptions are not necessarily alternatives; they could be different (dual) ways of looking at the same physics with the π

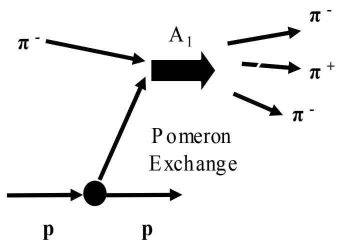


Fig. 2(a): Peripheral Production of A_1

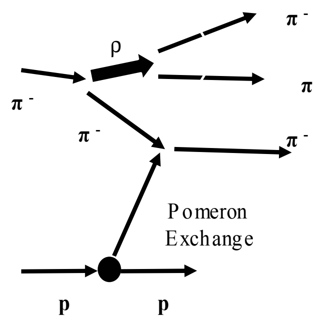
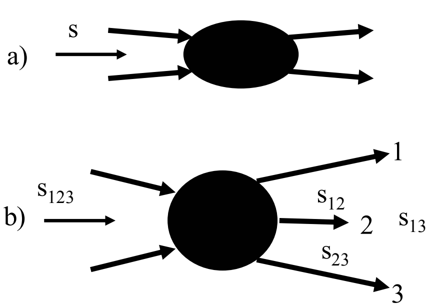


Fig. 2(b): Deck Model for A_1 3 π "enhancement"

exchange force producing the A_1 in elastic $\pi\,\rho$ scattering. Formulating duality as a precise rule is not easy. One good but incomplete way is to impose finite energy sum rules. Another less quantitative condition comes from quantum number connections between dual trajectories. The Pomeron and classic quark model exchanges are dual to different families of resonances (background for the Pomeron). Further for leading and daughter trajectories there are clear expectations of spin and other quantum numbers for Regge poles related by duality.

The constraint of unitarity is well understood mathematically but there are no straightforward ways of parameterizing amplitudes consistent with both unitarity and analyticity. In the two-body case (figure 3(a)), many formalisms such as the K matrix can guarantee unitarity but not analyticity and particular exchange particles. For the 3-body case, figure 3(b), Fadeev equations can guarantee unitarity in the sub channels but it is not clear this is "a good idea" as perhaps it is more important to respect analyticity as opposed to unitarity.



- **Impose unitarity in s for 2 particle PWA**
- **Impose unitarity in s_{12} s_{13} s_{23} for 3 particle PWA**
- **This is "final state interactions"**
- **Unitarity in s_{123} not a strong constraint**

Fig. 3: Direct Channel Constraints in a) 2 → 2 and b) 2 → 3 reactions

Such "conflicts" and formulations that express "part of the truth" are typical of S Matrix theory. There are similar "difficulties" in the spin formalism – namely in choosing the frames to be used for quantizing particle helicities. There are t-channel (reasonable for analyticity and respects exchanges), s-channel (best for analyticity) and transversity (best for selection rules but mixes amplitudes with different kinematic and hence analyticity factors.

We are looking at mesons M* produced by a mechanism similar to that of figure 4 with typically a multiparticle decay (here 3 particles). The A_1 production of Figure 2(a) is a special case of this. One will do better if this particular diagram is "big" as it would be if production exchange is one of the well understood trajectories with either a large coupling (π exchange) or an intercept $\alpha(0)$ that is higher than background exchanges. Pomeron ($\alpha(0) = 1$) and ρ, ω, A_2 ($\alpha(0) = 0.5$) fall into this second class. Even though this diagram is hard enough to analyze, there are many backgrounds such as that corresponding to diagrams like those of figure 5. Here one of the supposed decay products of the top vertex really comes from the bottom baryon vertex. Actually these are not necessarily distinct for multiperipheral diagrams like that of fig. 6 link the "bottom" and "top" vertex.

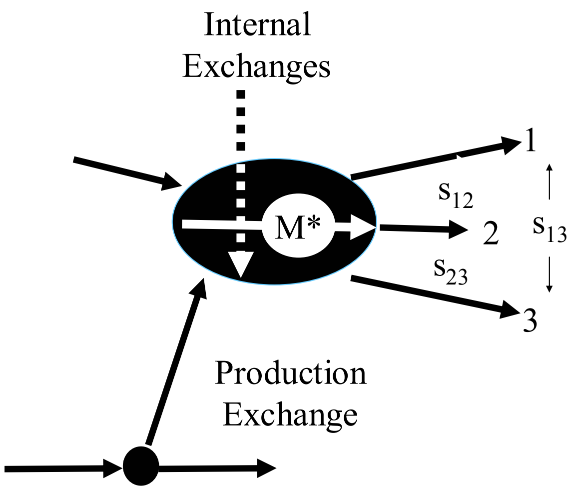


Fig. 4: Peripheral Production of a 3-body final state

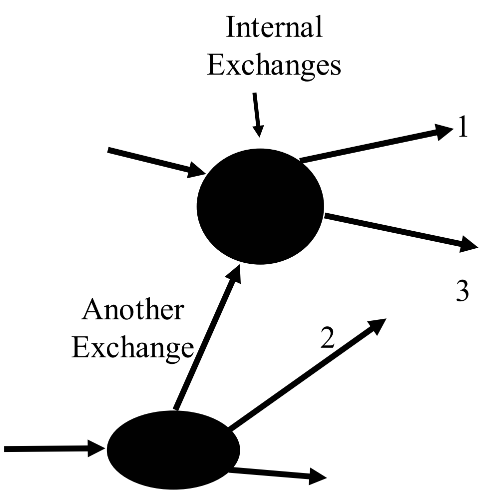


Fig. 5: Background from the "bottom" (baryon) vertex

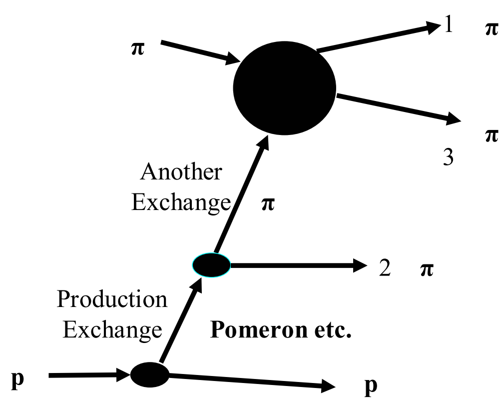


Fig 6: Multiperipheral Diagram corresponding to A_1 reaction of fig. 2.

This is labeled for A_1 production diagram but the role of multiperipheral diagrams is general. The intercept of the exchanged Reggeons in figure 6 is important; if as drawn the poles are the Pomeron and the π, then the "middle produced particle labeled 2" will be pushed to the top vertex. In contrast, if both exchanged links had the same intercept (*f* exchange for “production exchange” and A_2 for “another exchange” for example) then the middle produced particle 2 is truly equally associated with both vertices.

There are many other difficulties concerning unitarity. As mentioned above, we have ways of enforcing this but they do not respect crossing (duality) and analyticity very precisely in complicated cases. Unitarity suggests one looks at the rescattering diagrams like figure 7 where the rescatter can either be in “production process” or within final state decay or with some mixture. It is not well understood if the Reggeon expansion should be applied before or after “unitarization”. If one unitarizes a Regge amplitude, then one generates cuts in the angular momentum plane. However Reggeons are properties of the full amplitude and are not necessarily “Born terms” in some potential theory. Thus maybe the cuts generated by box diagrams such as those in figure 7 are deceptive. There is well known evidence suggesting cuts; examples include measured polarization in $\pi^- p \Rightarrow \pi^0 n$ and less convincingly phenomenology of π exchange. For partial wave analysis, diagrams such as those of figure 7 are extremely difficult to include in an amplitude parameterization. The $\pi^- p \Rightarrow \rho^0 n$ reaction is dominated by π exchange but measurements of the ρ density matrix elements violate the predictions of a naïve factorizable π exchange. One can trace difficulties to vanishing of helicity-flip $\pi \bar{p} n$ at t=0. The related reaction $\pi^+ p \Rightarrow \rho^0 \Delta^{++}$ has a very clean π exchange description compatible with simple Regge pole predictions. The features of the first reaction can be gotten by “absorption” (Regge cut) calculations but it is not easy to explain the normalization of the observed corrections. Instead a simple analyticity argument explains the data with proper normalization. This model (due to Williams) formally implies a companion pole (the conspirator) of opposite naturality in the angular momentum plane. It is not clear if the conspirator is truly a pole or rather as is perhaps more likely a phenomenological approximation to a different structure such as angular momentum plane cuts. The situation is sort of clear but frustrating. There are a set of “truths” which we do not understand how to make seamlessly compatible. In different reactions and different kinematic regions, one must agilely use the appropriate truth.

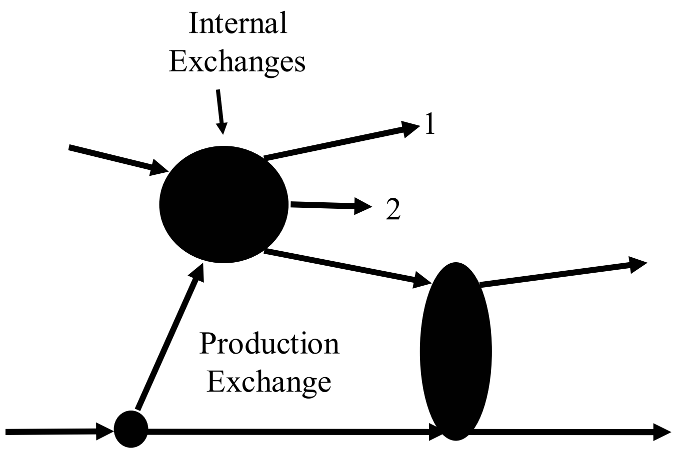


Fig 7: Rescattering Background

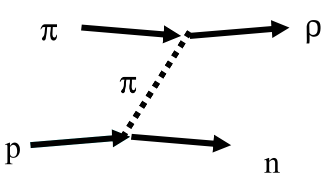


Fig 8: ρ production by π exchange

In summary, we have a set of basic truths and a phenomenology of how to assemble them to address any one reaction. Some “truths” are easier to apply and are more reliable than others. The quark model and QCD can give useful predictions as to both particle spectra and branching ratios. Selection rules are always good including those reflecting conservation of quark content. Regge theory has a few spectacular successes in describing high energy scattering. For example the linear ρ and A_2 exchange degenerate trajectories with α=0 dip for ρ and spin flip nucleon coupling typically behave largely as simple Regge poles should. Recent results on the Pomeron and indeed a deeper discussion of nearly all the issues discussed here can be found in [4]. Some areas like π exchange, polarization (sensitive to phases of the amplitude) and absorption (cuts) are more phenomelogical in their description. Some exchanges are known to be big or small and this phenomenology is uniformly successful. Finite energy sum rules can be used to express truths such as small high energy amplitudes for reactions like backward $\pi^+ \pi^-$ scattering) must translate to (amplitude) cancellation at low π π energies (ρ A_0 interference in the s channel)

In parameterizing amplitudes and fitting them to data (whether for PWA or otherwise), one must remember that it will be difficult to believe results sensitive to uncertainties in these "partial" theories.

Partial Wave Analysis
Let us see in more detail how the above "wisdom" can be applied to PWA. One can of course analyze production either in terms of a density matrix or an amplitude. The density matrix makes no assumptions but it is not possible to use it for detailed analysis to disentangle produced resonances. Thus most high statistics experiments expect to use an amplitude expansion. Amplitudes require assumptions but are the only easy way to express "truths" like analyticity; they also enforce rank and positivity conditions for the density matrix elements. The down side is that any realistic amplitude expansion makes assumptions and these could lead to biased results. The nature of our "truths" is that they do not allow amplitudes to be formulated in a way that allows increasing sophistication to be "switched on" by increasing the richness of parameterization. As already discussed we don't know how to simultaneously satisfy constraints. Rather one must make reasonable assumptions and check a posteriori how other "truths" are satisfied. Hardest of all one must estimate the impact of violations on ones conclusions. This requires multiple fits, good visualizations to examine validity of a given parameterization and new ways to synthesize them together. Just increasing number of partial waves is not in general a satisfactory approach as it doesn't directly translate into "satisfying truths".

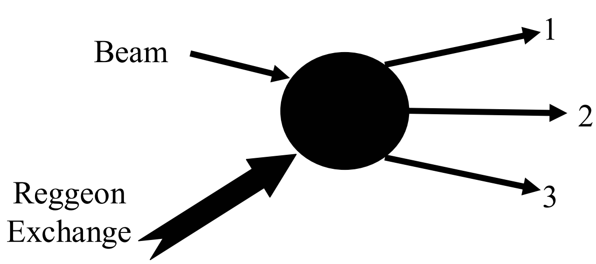


Fig. 9: Production as a Beam-Reggeon Scattering Amplitude

It is useful to consider the amplitude factorized so what you see corresponds to some reaction like Beam + Reggeon ⇨ 1 + 2 + 3 as shown in figure 9. Then the parameterized amplitudes depend on t (mass^2 of Reggeon) and helicities of beam, 1 2 and 3. This picture has obvious complications from multiple exchanges and cuts which would change expected form of final density matrix. One can expect this picture and indeed amplitude-based production PWA analysis to be most reliable when the underlying Reggeon phenomenology is convincing. This is the usual culprits: ρ ω and exchange degenerate A_2 f_2 families as well as the diffractive Pomeron are probably OK. The π exchange with adjustment for its conspirator should also be reliable. In contrast B_1 exchange (π exchange degenerate partner) is not expected to be very clean. Note the factorization picture says it doesn't matter what happens at the bottom vertex. Indeed ref. [2] showed that the same Regge ρ exchange picture describes $\pi^- p \Rightarrow \pi^0 n$, $\pi^- p \Rightarrow \pi^0$ inclusive, and $\pi^- p \Rightarrow \pi^0$ plus any neutral. It is perhaps useful to increase statistics by loosening vertex related trigger.

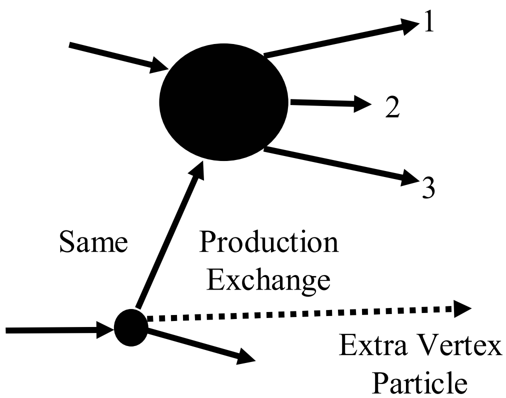


Fig. 10: Factorization at "Bottom" Vertex

In designing the PWA analysis process, we should have a framework that allows all "truths" to be examined and selected "truths" to be included explicitly. This requires attention to spin formalism, unitarity (rescattering), Regge exchange phenomenology and the PWA expansion and parameterization. In the spin formalism, one must consider:

- *Amplitude Parameterization* – is this practical if (potential) rank high as with photon beams and/or non-factorizing exchanges?
- *Density Matrix Formalism* – how does this cope with explicit contributions, analyticity etc.
- *Transversity versus helicity and s versus t* spin formalism needs to be considered – there is a trade-off of analyticity versus selection rules

For unitarity, one can use current approaches building in two-particle unitarity (final state interactions) but their integration with other "truths" is not easy. One should use both explicitly unitary and explicitly analytic (etc.) approaches and compare results. For a non-unitary parameterization, one can estimate the violation and examine the impact of this on the fit. When considering exchange contributions, one can identify all allowed (by normal Regge phenomenology) exchanges and catalog where they are expected to be large due to coupling constant values and/or values of α(t,u). One can use the usual duality type arguments to identify related s t and u channel exchanges i.e. find where you might expect the direct and crossed descriptions to be related. One should develop models for exchange contributions using simple phenomenological Regge theory but I would have little hope this is very reliable except for Pomeron, ρ and A_2 etc. exchanges but even these won't be so good as to be used quantitatively at low direct channel energies. One should identify all π exchange contributions and expect these to be reliable (with "conspirator" added) near t=0 but unreliable away from there. The status of π as a Regge pole is problematic. Coming to the PWA analysis itself, one can use Cutkosky style acceleration techniques to maximize convergence. This is certainly sound as it exploits fully analytic structure but possibly it is easier to explicitly include high partial waves rather than choose an expansion that maximizes convergence. This is particularly useful if we can estimate magnitude from known couplings at exchanged particle poles. Considering the Beam-Reggeon reaction of interest, this will often include internal exchanges such as the π exchange for which we have little experience from traditional πN direct channel analyses. Considering dispersion Relations and other analyticity constraints, we should certainly enforce "helicity dependent" kinematic zeroes. Further we should use visual inspection and finite energy sum rules to look at duality predictions. We could present data and fits in a way to highlight effects (e.g. we could look at fixed u cross sections for reactions like $\pi^+ \pi^-$ elastic scattering with no u channel exchanges).

Physics Summary

Unfortunately many confusing effects exist and there is no fundamental (correct) way to remove most of them. Theory has failed to provide convincing parameterizable amplitudes that one can use to fit/explain data. Rather the theory provides some quantitative constraints (π pole, unitarity, kinematics …), and many qualitative truths which overlap and whose effect can be estimated with errors from 10 to 100%. So one should try to minimize effect of the hard (insoluble) problems such as "particles from wrong vertex", "unestimatable exchange effects", sensitivity to slope of unclear Regge trajectories, absorption etc. One can note many of the effects (exchanges) are intrinsically more important in multiparticle case than in the well studied π N ⇨ π N reaction. We must try to estimate impact of uncertainties from each effect on results. This needs systematic very high statistic studies of relatively clean cases such as π N ⇨ π π N where spectroscopy may be less topical but where one can examine uncertainties. We need to first clarify impact of difficult issues and then apply this wisdom to new reactions and new meson resonances. One can be optimistic as we have orders of magnitude more data than when these issues were last looked at carefully.

Computational Issues in Analyzing PWA Data

We finish with a few remarks on computing issues associated with PWA analysis. The Grid techniques (http://www.grid2002.org) being developed for LHC physics will be useful but these largely address management of the data reduction problem. This is important but as not as critical as the difficult physics analysis discussed above. In considering PWA computing, we can perhaps assume unlimited computer time, disk space and network bandwidth or perhaps more precisely that there are no significant constraints from these issues.

One are of possible importance is the optimization (fitting) technique to match parameterized amplitudes and experiment. Actually physics has been ahead historically of the computer science field in area of multi-parameter fitting. It is possible that "unlimited computer time" and better management of results could lead to new approaches such as "ensemble fitting techniques" (e.g. genetic algorithms) which explore parameter space more completely than traditional (Taylor expansion) least squares methods. We need to develop techniques that are not specially aimed at finding "new/best solutions" but rather on better depicting error band of a given "solution". For example we could have "histograms" which automatically display ensemble and not a single "best fit" – we have plenty of memory/disk space to store the data to be displayed. More generally, we can think of our problem as data mining where we need to develop a suite of analysis programs that look for and catalog anomalies in experiment, theory or the discrepancy between them. We need to link this to the data management system so as to more systematically categorize different fits. Perhaps we could develop new multi-dimensional visualization (human data-mining) methods that improve dramatically on traditional histograms. Perhaps we could develop some better "standard" displays supported in a modern computer approach as a Web Services with portlet interfaces (this suggestion betrays my current expertise). Here the printed version of Phys. Rev. Letters still has something like a histogram playing role of an "icon", which is active in on-line version and can access the "new nifty display".

In summary we can exploit modern computing technology for more reliable optimization methods, the management of data before (as in GriPhyn and PPDG [5]) and especially in our case after analysis. We should explore powerful multi-dimensional visualization techniques and data mining to discover anomalies in data and/or fits and/or discrepancies between fits and data. We need to design and build such an environment and as part of the physics imperative apply to relatively well understood reactions to clarify the difficult conflicting physics issues.

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