Qualifying Exam

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Abstract

In order to satisfy qualifying exam requirements adhered to by the School of Informatics and Computing, I will provide an overview of global optimization techniques, specifically simulated annealing, genetic algorithms, and tabu seach.

Introduction

An optimization problem involves a set of candidate solutions, S, and an objective function, which measures the quality of a solution. In general, the search space, S, is defined implicitly and consists of a number of candidate solution. When the objective function determines the quality of a solution, it is referred to as an energy minimization problem, and E is called an energy function. The ideal solution to an optimization problem would be a candidate solution, which is a global minimal of the energy function. The following discuss a few global optimization techniques.

Simulated Annealing

Simulated annealing [1], which was inspired by the natural process of annealing solids. The physical process of annealing involves slowing cooling metal, so it adopts a low energy, crystalline state. When the temperature of the metal is high, the particles within the metal are active, changing the metal structure. As the temperature is lowered, the particles are limited in movement since a high energy cost are very limited to

configurations with a lower energy. Simulated annealing uses the idea of a physical process, in a computational model. The basic algorithm maintains both a state and a temperature, which is initially high and is reduced to near zero according to a cooling schedule. The configuration is typically a solution to the optimization, and at each iteration of the algorithm, this solution is changed to produce a new solution. The quality of the solution is evaluated using the objective function, and a new state is selected from the two solutions. When a new solution is better than the previous, the new solution is chosen, but when the new solution has a lower quality than the existing solution, it may be accepted with a probability depended on the current temperature and the difference in quality. With certain cooling schedules, annealing can be guaranteed to find a global minimum.

Genetic Algorithm

A genetic algorithm [2], which was proposed by Holland [3] in his 1975 paper, is search technique based upon principles of genetics and natural selection. Genetic algorithms allow a population composed of many chromosomes, which is a unique solution to the problem, to evolve until the population includes better solutions and converges, into a single solution. Of the three operators for generating new solutions crossover and mutation, are the most popular. In the crossover method, two chromosomes, called parents, are combined to form new chromosomes, called offspring. The parents are selected among existing chromosomes in the population with preference towards fitness, so offspring is expected to inherit good genes, which make the parents fitter. By iteratively applying the crossover operator, genes of good chromosome are expected to

appear more frequently in the population, eventually leading to convergence to an overall good solution. The mutation operator introduces random changes into characteristics of chromosomes. Mutation reintroduces genetic diversity back into the population and assists the search escape from local optima. Reproduction involves selection of chromosomes for the next generation. In most cases, the fitness of an individual determines the probability of its survival for the next generation.

Tabu Search

Tabu search was pioneered by Glover published many articles discussing its numerous applications. Others works using tabu search include: sequencing [4], scheduling [5, 6, 7, 8], oil exploration [9] and routing [10]. The properties of the tabu search can be used to enhance other procedures by preventing them from becoming limited in regions of local minima. The tabu search introduces memory structures by preventing the search from returning to a previously explored region of the solution space. This is achieved by retaining a list of possible solutions, which have been previously encountered. The size of the tabu list is one of the parameters of the tabu search. The tabu search also contains mechanisms for controlling the search. The tabu list ensures some solutions will be unacceptable, however, the restriction provided by the tabu list may become too limiting in cases causing the algorithm to become trapped at a locally optimum solution. The tabu search introduces the notion of aspiration criteria in order to overcome this problem. The aspiration criteria override the tabu restrictions making it possible to broaden the search for the global optimum. The tabu list is initialized with the initial solution. A number of iterations are performed which attempts to update the current solution with a better one,

subject to the restrictions of the tabu list. In each iteration, a list of candidate solutions is proposed. The current solution is updated with the most admissible one and the new current solution is added to the tabu list. The algorithm stops after a fixed number of iterations or when a better solution has been found for a number of iterations.

Conclusion

Simulated annealing mimics the process of annealing in metals similar to the structure of molecules of the heated metal. When the temperature is high, the molecules move at random and appear to have small order. This may represent an initial random guess at a solution to an optimization problem. After some time, as the temperature slowly cools, the molecules move toward a more ordered structure, the aim of annealing being to produce a crystalline structure in the molecules. The analogy to optimization is still present so the algorithm progresses to a more ordered solution are obtained. The genetic algorithm is an attempt to use Darwin's evolutionary model in the field of optimization, which has proven to be remarkably successful. A pool of solutions breed, and mutate in a survival of the fittest regime. Solutions not considered suitable die off so that, over time, the solution pool contains good solutions to the problem. Initially intended to operate on problems whose solutions could be represented as a binary string, the genetic algorithm has grown to cover the field known as "evolutionary programming" where an arbitrary solution representation can be utilized, provided suitable genetic operators can be created. The tabu search incorporates techniques for ensuring that the solutions considered in the search are diverse. This is achieved by maintaining a tabu list which contains a list of solutions which have been visited by the search previously and may not be accepted again, at least not until a certain amount of time has passed. However, by specifying

aspiration criteria, the tabu list can be overridden in order to ensure solutions, which are believed to be good.

References

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