

# Analysis for Optimal Degree of Replication

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**Abstract.** Replication technologies are widely used in distributed systems to improve performance in terms of availability and scalability. We analyze cost of replication in various aspects, which help us to choose proper degree of replication in error-prone systems. Our proposed cost model and algorithm to choose proper degree of replication is simple. However, it would help us to decide whether or not to replicate and, if replicate, degree of replication in our system design.

**Keywords:** Distributed Systems; Data Grid; Data Replication; Reliability; Fault-tolerance

## 1 Introduction

Replication technologies are widely used in distributed systems to improve performance in terms of availability and scalability. Many system components (e.g., disk storage, file systems, databases, software tasks, and system itself) can be replicated to obtain the advantages. In fault-prone environments, replication strategies of system components (hardware or software) are popularly adopted to improve system availability [2, 5, 6]. Many replication methods [1, 3 - 6] were proposed to increase availability and scalability. Some researches [1, 3, 4] were performed to obtain proper degree or location of replication to reduce total access cost by considering trade-off between cost for resource access and cost for component replication (e.g., cost of component or maintaining consistency among components). However, few researches were investigated to obtain proper degree of replication to minimize total costs in error-prone environments.

We need to consider many aspects to decide replications and degree of it; cost of physical component, re-do (or recovery) overhead on a failure, failure rate, and access (read and write) rate and cost. We analyze the cost of replication in various aspects, which help us to choose proper degree of replication in error-prone systems.

## 2 Cost Analysis for Degree of Replication

The total cost of replication is:

$$c_s(x) = c_p(x) + c_a(x) + c_r(x),$$

where  $c_s(x)$  is total cost function,  $x$  is degree of replication,  $c_p(x)$  is cost of physical replication,  $c_a(x)$  is cost of resource access, and  $c_r(x)$  is cost of re-do overhead due to all  $x$  replications' failures.

Now, we analyze the three costs:  $c_p(x)$ ,  $c_a(x)$ , and  $c_r(x)$ .

- (1) We assume that cost of physical replication is proportional to the number of replicated component:

$$c_p(x) = xc_{phy}, \text{ where } c_{phy} \text{ is cost of physical component per time unit.}$$

- (2) We assume that cost of access consists of read cost and write cost (consistency overhead among replications). In general, read cost adversely proportional to the number of replication while consistency overhead is proportional to:

$$c_a(x) = \lambda_{read} \frac{c_{read}}{x} + \lambda_{write} xc_{consist},$$

where  $\lambda_{read}$ ,  $\lambda_{write}$ ,  $\frac{c_{read}}{x}$ , and  $xc_{consist}$  denote read rate, write rate, read overhead, and consistency overhead, respectively.

- (3) Re-do overhead is required on all  $x$  replicated components' failures:

$$c_r(x) = (1 - e^{-\lambda T})^x c_{re-do},$$

where  $\lambda$ ,  $T$ , and  $c_{re-do}$  are failure rate, transaction time, and re-do overhead, respectively.

$$\text{Thus, } c_s(x) = c_p(x) + c_a(x) + c_r(x)$$

$$= xc_{phy} + \lambda_{read} \frac{c_{read}}{x} + \lambda_{write} xc_{consist} + (1 - e^{-\lambda T})^x c_{re-do}$$

We analyze replication cost as the function of replication degree. Our analysis is simple but it is useful to decide proper replication degree in the various applications in which condition is different to each other.

In general, failure rate is small. Thus, we can approximate  $c_s(x)$  as follows:

$$c_s(x) = xc_{phy} + \lambda_{read} \frac{c_{read}}{x} + \lambda_{write} xc_{consist} + (1 - e^{-\lambda T})^x c_{re-do}$$

$$\cong xc_{phy} + \lambda_{read} \frac{c_{read}}{x} + \lambda_{write} xc_{consist} + (\lambda T)^x c_{re-do} \quad (1)$$

If we assume that access cost ( $c_{access}$ , read and write cost) is constant ( $c_a$ ) or ignored ( $c_a = 0$ ), then;

$$c_s(x) = xc_{phy} + c_a + (\lambda T)^x c_{re-do} \quad (2)$$

$$\rightarrow \frac{\partial c_s}{\partial x} = c_{phy} + (\lambda T)^x \ln(\lambda T) c_{re-do}$$

$$\text{As } \frac{\partial c_s}{\partial x} = 0 \text{ when } x = \ln_{\lambda T} \left( -\frac{c_{phy}}{\ln(\lambda T) c_{re-do}} \right),$$

$c_s(x)$  is minimum when  $x = \ln_{\lambda T} \left( -\frac{c_{phy}}{\ln(\lambda T)c_{rs-do}} \right)$

Now, we obtain the optimal solution of  $x$  (degree of replication).  $c_s(x)$ , total cost function of  $x$  (degree of replication), has the minimum value,

$$\ln_{\lambda T} \left( -\frac{c_{phy}}{\ln(\lambda T)c_{rs-do}} \right) c_{phy} + c_a - \frac{c_{phy}}{\ln(\lambda T)c_{rs-do}} c_{rs-do},$$

when  $x = \ln_{\lambda T} \left( -\frac{c_{phy}}{\ln(\lambda T)c_{rs-do}} \right)$ .

### 3 Conclusion

Replication is widely used technology to increase availability and scalability. However, it requires costs for physical redundancy and consistency maintenance among replications. We analyzed the cost of replication to choose the proper degree of replication according to various system parameters: cost of physical component, re-do overhead, and failure rate. Our analysis is simple. But, we can select the optimal degree of replication on a system design considering such various aspects. In general, the lower cost of physical component, the higher re-do overheads, and high failure rate encourages high degrees of replication.

**Acknowledgments.** This research is supported by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education, Science and Technology (No. 2011-0027263). Also, this research is supported by the MKE (Ministry of Knowledge Economy), Korea, under the "Employment Contract based Master's Degree Program for Information Security" supervised by the KISA (Korea Internet Security Agency).

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